FIR Model Identification of Multirate Processes with Random Delays Using EM Algorithm

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The motivation for this article comes from our development of soft sensors for chemical processes where several challenges are encountered. For example, quality variables in chemical processes are often measured off-line through laboratory analysis. Collection of samples and subsequent analyses inevitably introduce uncertain time delays associated with the irregularly sampled quality variables, which add significant difficulty in identification of process with multirate (MR) data. Considering the MR system with random sampling delays described by a finite impulse response (FIR) model, an Expectation–Maximization (EM)-based algorithm to estimate its parameters along with the time delays is developed. Based on the identified FIR model, two algorithms are proposed to recover the approximate output error (OE) or transfer function model. Two simulation examples as well as a pilot-scale experiment are provided to illustrate the effectiveness of the proposed methods. © 2013 American Institute of Chemical Engineers *AIChE J*, 59: 4124–4132, 2013

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Introduction

Multirate (MR) systems arise often in typical chemical and bio-chemical processes due to absence of online measurements for certain quality variables.[1,2] For example, the distillate and bottom compositions in distillation columns are usually sampled infrequently through off-line laboratory analysis, while other process variables such as tray temperature, flow rate, pressure, and level are readily measured at fast rate.[3] Real-time information about quality variables is essential to effective process monitoring, control, and optimization. Therefore, various methods have been proposed to model the MR systems and infer the unmeasurable or missing outputs.

First principle models are preferred for MR system modeling because these models are obtained from fundamental process knowledge and have clear physical interpretation. However, this kind of models are not always available due to complexity and lack of thorough understanding of the process.[4] In contrast, data-driven models are developed based on process operational data, which are less dependent on prior process knowledge.[5] Additionally, data-driven models are convenient to implement on the existing distributed control systems. Despite its limited extrapolation ability, data-driven modeling has received increasing attention in recent years.

One of the most frequently used techniques to build data-driven models from MR sampled data is to down-sample the process data in accordance with the slow sampling rate of the quality variables.[6] Based on this technique, Kano et al. (2003)[7] developed a dynamic partial least-squares model to predict the bottom composition of a pilot-scale distillation column using online measurements of process variables. The predicted composition was treated as controlled variables, and a predictive inferential controller was further implemented to improve the product control performance. Shakil et al. (2009)[8] also employed the down-sampling technique and presented a dynamic artificial neural network model to estimate NO_x and O_2 of an industrial boiler, where principle component analysis and genetic algorithm were adopted to reduce the process data dimension and to analyze the time delays, respectively. Although down-sampling technique is straightforward to implement in practice, it has a critical drawback of information loss and may lead to inaccurate

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models when the quality variables are sampled scarcely with uncertain delays.

Lifting technique is another fundamental tool for analysis of MR systems. The basic idea is to rearrange the MR input–output data to obtain an equivalent slow-rate state-space model with increased dimensionality.[9] Typically, conventional identification methods for single-rate multivariable systems are applicable to the lifted models, but the causality constraint problem must be appropriately considered. Ding and Chen (2005)[10] decomposed the lifted state-space model into several subsystems to tackle the causality constraint, and proposed a hierarchical identification method for combined parameter and state estimation of MR systems. In addition to causality constraint, MR controllers based on the lifted model may lead to inter-sample ripples in the controlled outputs. To avoid this problem, Li et al. (2001, 2003)[11,12] investigated a computational method to extract the fast-rate state-space model from the lifted one for estimation of the inter-sample outputs, and presented an inferential control algorithm for MR systems.

Polynomial transformation technique is similar to the lifting technique, which is facilitated to transform the fast-rate transfer function model into an equivalent form that does not require the missing output data in identification. However, this technique only applies to special MR systems, i.e., dual-rate systems, where the output sampling period is an integer multiple of the input updating period.[13] Furthermore, the transformed model involves much more parameters than the original one, resulting in a nontrivial computational load of the corresponding identification algorithm. A more efficient alternative to this approach is to identify the fast-rate model directly from the MR data by means of auxiliary model identification principle.[14] The basic idea is to establish an auxiliary model using the finite impulse response (FIR) model or the estimated fast-rate model and use its outputs to approximate the unmeasurable variables, enabling traditional single-rate identification methods to be used for MR systems.[15] The auxiliary model-based approach is also applicable to nonuniformly sampled MR systems, but it needs to assume the associated noise to be white.[16]

Identification of MR systems with irregular output sampling has received increasing attention in recent years.[16–19] For linear systems, Raghavan et al. (2006)[17] combined the expectation–maximization (EM) algorithm with Kalman filter to study state-space model identification from MR data in the presence of irregularly sampled outputs. The EM-based identification approach was further extended to nonlinear MR systems under irregular missing observations, where particle filter was adopted to approximate the density functions required in the expectation step.[18] In practice, the irregularly sampled outputs are often available with random delays due to manual analysis in laboratory. Moreover, the associated sampling delays are uncertain since only the arrival time of the measurements is recorded, while the time instant that the samples are actually taken is unknown or not accurately recorded. Such a delay issue can significantly increase the difficulty in identification of MR systems, and lead to inaccurate parameter estimates if not handled appropriately. To resolve this problem, this article will derive an EM-based algorithm to identify the MR systems, taking into account uncertain random delays associated with the irregularly sampled outputs.

The problem discussed in this article is more complicated and challenging than the one in Ref. 17 because they did not consider about the uncertain random delay issue. In addition, input–output representation is utilized in this article to describe the MR systems, which contains fewer number of parameters to be estimated than a noncanonical state-space model used in Ref. 17. Accordingly, the computational complexity has been reduced since Kalman filter is no longer required to predict the state and output trajectory.

The remainder of this article is organized as follows. The section entitled Expectation–Maximization (EM) algorithm gives a brief introduction to the EM algorithm. The following section states the FIR model identification problem of MR systems in the presence of uncertain random delays. The next section applies an EM algorithm to solve this problem. The section Estimation of MR OE Model extends the proposed EM algorithm to identify the MR output error models with random sampling delays based on the identified FIR model. Two simulation examples as well as a pilot-scale experiment are presented in the section titled Simulations for validation of the proposed methods, followed by the Conclusions section.

Expectation–Maximization (EM) algorithm

The EM algorithm was proposed by Dempster et al. (1977)[20] for computation of maximum likelihood estimates from incomplete datasets. It consists of an Expectation step (E-step) and a Maximization step (M-step), where the E-step calculates the expectation of the complete data log-likelihood function (Q function) with respect to the unobserved hidden variables based on the previously estimated parameter vector, and the M-step obtains a new estimate of parameter vector through maximizing the Q function. These two steps are iteratively operated until the parameter estimates converge.

Assume a complete dataset $\mathscr{L}$ that comprised an observed part $\mathscr{Y}$ and an unobserved part $\mathscr{X}$. Let $h=0$ and Θ^0 be an initial guess of model parameter Θ; the mathematical formulation of EM algorithm can be expressed as[21]:

- E-step: Given Θ^h estimated from the previous iteration, the Q function is calculated by

$$\begin{aligned} Q(\boldsymbol{\Theta}|\boldsymbol{\Theta}^h) &= \mathrm{E}_{\mathscr{X}|\mathscr{Y},\boldsymbol{\Theta}_h}\{\log[p(\mathscr{Y},\mathscr{X}|\boldsymbol{\Theta})]\} \\ &= \int \log[p(\mathscr{Y},\mathscr{X}|\boldsymbol{\Theta})]p(\mathscr{X}|\mathscr{Y},\boldsymbol{\Theta}^h)\mathrm{d}\mathscr{X} \end{aligned}$$

- M-step: Maximize the Q function with respect to $\boldsymbol{\Theta}$ to obtain the new iterative parameter estimate $\boldsymbol{\Theta}^{h+1}$

$$\boldsymbol{\Theta}^{h+1}=\arg\max Q(\boldsymbol{\Theta}|\boldsymbol{\Theta}^h)$$

- Evaluate the relative change of parameter estimates

$$\delta_{h+1}=\frac{\|\boldsymbol{\Theta}^{h+1}-\boldsymbol{\Theta}^h\|^2}{\|\boldsymbol{\Theta}^h\|^2}$$

If it is larger than a predetermined tolerance ε, then let $h=h+1$ and repeat Steps 1 and 2.

The EM algorithm has become a popular computational tool in Statistics, which has found various applications in system modeling, computer vision, pattern recognition, and machine learning. Its convergence property under general conditions was established by Wu (1983).[22]

Problem Formulation

Consider the following MR FIR model

$$x_k=\left(f_0+f_1z^{-1}+f_2z^{-2}+\cdots+f_{n_f}z^{-n_f}\right)u_k \tag{1}$$

$$y_{T_i}=x_{T_i-\lambda_i}+v_{T_i} \tag{2}$$

where $\{u_k, k=1,2,\cdots,L\}$ is the input and available at every sampling period Δt; $\{x_k\}$ is the unmeasurable noise-free output; $\{y_{T_i}, i=1,2,\cdots,N\}$ is the irregularly sampled output and only available at time instant $t=T_i\cdot\Delta t$ with unknown time delay $t_{d_i}=\lambda_i\cdot\Delta t$ (i.e., the delay can vary in each data sample); v_{T_i} is the associated measurement noise with unknown Gaussian distribution $N\left(0,\sigma_v^2\right)$. For the sake of simplicity, we assume that the system order n_f is known. The delay λ_i is a random integer that can follow any discrete distribution. In this article, it is assumed to be uniformly distributed between 0 and q, i.e.,

$$p(\lambda_i=j)=\frac{1}{q+1},\quad j=0,1,\ldots,q \tag{3}$$

Our objective is to estimate the parameters $\{f_j, j=0,1,2,\ldots,n_f\}$ in Eq. 1 along with the delays $\{\lambda_i, i=1,2,\ldots,N\}$ and the variance σ_v^2 of measurement noise v_{T_i} in Eq. 2 based on the available MR data $\{u_k\}$ and $\{y_{T_i}\}$. However, the outputs $\{y_{T_i}\}$ are involved with uncertain random delays $\{\lambda_i\}$, which significantly increases the difficulty to identify the underlying system.

MR FIR Model Identification using the EM Algorithm

Define the information vector $\boldsymbol{\psi}_k$ and the parameter vector $\boldsymbol{\vartheta}$ as

$$\boldsymbol{\psi}_k=\left[u_k,u_{k-1},u_{k-2},\cdots,u_{k-n_f}\right]\in\mathbb{R}^{1\times\left(n_f+1\right)}$$

$$\boldsymbol{\vartheta}=\left[f_0,f_1,f_2,\cdots,f_{n_f}\right]^{\mathrm{T}}\in\mathbb{R}^{\left(n_f+1\right)\times 1}$$

Eq. 1 can be written into the following vector form

$$x_k=\boldsymbol{\psi}_k\boldsymbol{\vartheta} \tag{4}$$

Substituting Eq. 4 into Eq. 2 yields

$$y_{T_i}=\boldsymbol{\psi}_{T_i-\lambda_i}\boldsymbol{\vartheta}+v_{\mathbf{T}_i} \tag{5}$$

Let $[\boldsymbol{\Theta}=\left[\boldsymbol{\vartheta};\sigma_{\boldsymbol{\omega}}^2\right]$ represent the overall parameters to be identified, $\mathscr{U}=u_{1:L}$ denote the sequence of input $\{u_1,u_2,\cdots,u_L\}$, $\mathscr{Y}=y_{T_1:T_N}$ denote the observed output $\{y_{T_1},y_{T_2},\cdots,y_{T_N}\}$, and $\boldsymbol{\Gamma}=\lambda_{1:N}$ stand for the random sampling delay $\{\lambda_1,\lambda_2,\cdots,\lambda_N\}$ of all the observed output.

Considering unknown delays $\lambda_{1:N}$ as the hidden states, the Q function can be determined as the expectation of the log-likelihood function $\log\left[p(\mathscr{U},\mathscr{Y},\Gamma|\boldsymbol{\Theta})\right]$ with respect to all the hidden states, given by

$$Q\left(\Theta|\Theta^h\right)=\mathrm{E}_{\Gamma|\mathscr{U},\mathscr{Y},\Theta^h}\{\log\left[p(\mathscr{U},\mathscr{Y},\Gamma|\Theta)\right]\} \tag{6}$$

where $\boldsymbol{\Theta}^h$ is the estimate of $\boldsymbol{\Theta}$ after hth iteration. Using the Bayesian theorem, the joint probability density function $p(\mathscr{U},\mathscr{Y},\Gamma|\boldsymbol{\Theta})$ that is needed to compute the expectation in Eq. 6 can be derived as

$$\begin{aligned}p(\mathscr{U},\mathscr{Y},\Gamma|\Theta)&=p(\mathscr{Y}|\mathscr{U},\Gamma,\Theta)p(\mathscr{U},\Gamma|\Theta)\\&=p(\mathscr{Y}|\mathscr{U},\Gamma,\Theta)p(\Gamma|\mathscr{U},\Theta)p(\mathscr{U}|\Theta)\end{aligned} \tag{7}$$

According to Eq. 5, y_{T_i} is only dependent on λ_i, $\boldsymbol{\psi}_{T_i-\lambda_i}$ and $\boldsymbol{\Theta}$. Therefore, we have

$$p(\mathscr{Y}|\mathscr{U},\Gamma,\boldsymbol{\Theta})=\prod_{i=1}^{N}p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-\lambda_i},\lambda_i,\boldsymbol{\Theta}\right) \tag{8}$$

Because the occurrence of time delay is completely random and does not depend on the inputs and model parameters, thus

$$p(\Gamma|\mathscr{U},\boldsymbol{\Theta})=\prod_{i=1}^{N}p(\lambda_i) \tag{9}$$

Since input $\mathscr{U}$ is known and independent of $\boldsymbol{\Theta}$, the last term in Eq. 7 should be a constant C. Using Eqs. 8–9 and substituting Eq. 7 into Eq. 6, the Q function can be derived as

$$\begin{aligned}Q\left(\Theta|\Theta^h\right)&=\mathrm{E}_{\Gamma|\mathscr{U},\mathscr{Y},\Theta^h}\left\{\log\left[C\prod_{i=1}^{N}p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-\lambda_i},\lambda_i,\Theta\right)p(\lambda_i)\right]\right\}\\&=\mathrm{E}_{\Gamma|\mathscr{U},\mathscr{Y},\Theta^h}\left\{\log C+\sum_{i=1}^{N}\log\left[p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-\lambda_i},\lambda_i,\Theta\right)\right]+\log\left[p(\lambda_i)\right]\right\}\end{aligned} \tag{10}$$

By moving the Expectation operator inside the summation, Eq. 10 becomes

$$\begin{aligned}Q\left(\Theta|\Theta^h\right)&=\log C+\sum_{i=1}^{N}\mathrm{E}_{\Gamma|\mathscr{U},\mathscr{Y},\Theta^h}\left\{\log\left[p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-\lambda_i},\lambda_i,\Theta\right)\right]+\log\left[p(\lambda_i)\right]\right\}\\&=\log C+\sum_{i=1}^{N}\sum_{j=0}^{q}p\left(\lambda_i=j|\mathscr{U},\mathscr{Y},\Theta^h\right)\log\left[p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-\lambda_i},\lambda_i=j,\Theta\right)\right]\\&\quad+\sum_{i=1}^{N}\sum_{j=0}^{q}p\left(\lambda_i=j|\mathscr{U},\mathscr{Y},\Theta^h\right)\log\left[p(\lambda_i=j)\right]\end{aligned} \tag{11}$$

where $p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-\lambda_i},\lambda_i=j,\boldsymbol{\Theta}\right)$ is calculated by

$$p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-\lambda_i},\lambda_i=j,\boldsymbol{\Theta}\right)=\frac{1}{\sqrt{2\pi}\sigma_v}\exp^{-\frac{1}{2\sigma_v^2}\left[y_{T_i}-\boldsymbol{\psi}_{T_i-j}\boldsymbol{\vartheta}\right]^2} \tag{12}$$

and accordingly

$$\log\left[p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-\lambda_i},\lambda_i=j,\boldsymbol{\Theta}\right)\right]=-\log\sqrt{2\pi}\sigma_v-\frac{1}{2\sigma_v^2}\left[y_{T_i}-\boldsymbol{\psi}_{T_i-j}\boldsymbol{\vartheta}\right]^2 \tag{13}$$

For notation simplicity, w_{ij} is used to represent $p\left(\lambda_i=j|\mathscr{U},\mathscr{Y},\boldsymbol{\Theta}^h\right)$ in the remainder of this article. It denotes the conditional probability that the delay of the ith measured output y_{T_i} equals to j, and can be derived using the Bayesian rule as

$$\begin{aligned}w_{ij}&=p\left(\lambda_i=j|\mathscr{U},\mathscr{Y},\boldsymbol{\Theta}^h\right)=p\left(\lambda_i=j|y_{T_i},\boldsymbol{\psi}_{T_i-j},\boldsymbol{\Theta}^h\right)\\&=\frac{p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-j},\lambda_i=j,\boldsymbol{\Theta}^h\right)p\left(\lambda_i=j|\boldsymbol{\psi}_{T_i-j},\boldsymbol{\Theta}^h\right)}{\sum_{j=0}^{q}p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-j},\lambda_i=j,\boldsymbol{\Theta}^h\right)p\left(\lambda_i=j|\boldsymbol{\psi}_{T_i-j},\boldsymbol{\Theta}^h\right)}\\&=\frac{p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-j},\lambda_i=j,\boldsymbol{\Theta}^h\right)p(\lambda_i=j)}{\sum_{j=0}^{q}p\left(y_{T_i}|\boldsymbol{\psi}_{T_i-j},\lambda_i=j,\boldsymbol{\Theta}^h\right)p(\lambda_i=j)}\end{aligned} \tag{14}$$

Substituting Eq. 13 into Eq. 11 and replacing $p\left(\lambda_i=j|\mathscr{U},\mathscr{Y},\boldsymbol{\Theta}^h\right)$ by w_{ij}, the Q function is finally derived as

$$Q(\Theta|\Theta^h)=\log C+\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}\times\left[-\log\sqrt{2\pi}\sigma_v-\frac{1}{2\sigma_v^2}\left[y_{T_i}-\boldsymbol{\psi}_{T_i-j}\boldsymbol{\vartheta}\right]^2\right]+\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}\log\left[p(\lambda_i=j)\right] \quad (15)$$

Note that $p(\lambda_i=j)$ is given by Eq. 3. Identification of Θ using the EM algorithm can be summarized as follows:

- **E-step:** Calculate $w_{ij}, j=0,1,\cdots,q$ based on Eq. 14, where $p(y_{T_i}|\boldsymbol{\psi}_{T_i-j},\lambda_i=j,\Theta^h)$ is computed based on Eq. 12 using the old parameter estimate $\boldsymbol{\vartheta}^h$ and $(\sigma_v^2)^h$; then substitute w_{ij} into Eq. 15 to calculate the approximate Q function. If the point estimation of delays is desired, using the maximum *a posteriori* principle, the delay estimate $\hat{\lambda}_i$ can be determined by searching for the largest w_{ij} among all the possibilities of delay, i.e.,

$$\hat{\lambda}_i=\arg\max_j w_{ij}, \quad j=0,1,\cdots,q$$

- **M-step:** Taking derivative of the Q function in Eq. 15 with respect to $\boldsymbol{\vartheta}$ and σ_v^2, respectively, yields

$$J(\boldsymbol{\vartheta})=\frac{\partial Q(\Theta|\Theta^h)}{\partial\boldsymbol{\vartheta}}=\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}\left[-\frac{1}{\sigma_v^2}\boldsymbol{\psi}_{T_i-j}^{\mathrm{T}}\left[y_{T_i}-\boldsymbol{\psi}_{T_i-j}\boldsymbol{\vartheta}\right]\right]$$

$$J(\sigma_v^2)=\frac{\partial Q(\Theta|\Theta^h)}{\partial\sigma_v^2}=\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}\left[-\frac{1}{2\sigma_v^2}+\frac{1}{2\sigma_v^4}\left[y_{T_i}-\boldsymbol{\psi}_{T_i-j}\boldsymbol{\vartheta}\right]^2\right]$$

To maximize the Q function, $J(\boldsymbol{\vartheta})$ and $J(\sigma_v^2)$ are set to zero and the new parameter estimate $\boldsymbol{\vartheta}^{h+1}$ and $(\sigma_v^2)^{h+1}$ can be calculated by

$$\boldsymbol{\vartheta}^{h+1}=\left\{\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}*\boldsymbol{\psi}_{T_i-j}^{\mathrm{T}}\boldsymbol{\psi}_{T_i-j}\right\}^{-1}\left\{\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}*\boldsymbol{\psi}_{T_i-j}^{\mathrm{T}}y_{T_i}\right\} \quad (16)$$

$$(\sigma_v^2)^{h+1}=\left\{\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}\right\}^{-1}\left\{\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}*\left[y_{T_i}-\boldsymbol{\psi}_{T_i-j}\boldsymbol{\vartheta}^{h+1}\right]^2\right\}=\frac{1}{N}\sum_{i=1}^{N}\sum_{j=0}^{q}w_{ij}*\left[y_{T_i}-\boldsymbol{\psi}_{T_i-j}\boldsymbol{\vartheta}^{h+1}\right]^2 \quad (17)$$

These two steps are iterated until the parameter estimate $\boldsymbol{\vartheta}^h$ and $(\sigma_v^2)^h$ converge to certain value $\hat{\boldsymbol{\vartheta}}=[\hat{f}_0,\hat{f}_1,\hat{f}_2,\cdots,\hat{f}_{n_f}]^{\mathrm{T}}$ and $\hat{\sigma}_v^2$, respectively.

Estimation of MR OE Model

Output error (OE) or transfer function models have been widely used to design advanced control. A MR FIR model with sufficiently large order n_f in Eqs. 1 and 2 can be used to approximate the following stable MR OE model given by Eqs. 18 and 19

$$x_k=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots+b_{n_b}z^{-n_b}}{1+a_1z^{-1}+a_2z^{-2}+\cdots+a_{n_a}z^{-n_a}}u_k \quad (18)$$

where u_k is the input, x_k is the noise-free output, n_a and n_b are the known system orders. The output measurement equation is the same as Eq. 2, i.e.,

$$y_{T_i}=x_{T_i-\lambda_i}+v_{T_i} \quad (19)$$

The relationship between the parameters of OE model 18 and FIR model 1 is described as

$$\begin{cases} f_0=b_0, \\ f_j=b_j-\sum_{l=0}^{j-1}a_{j-l}\cdot f_l, \quad \text{for } j=1,2,\cdots,n_f \end{cases} \quad (20)$$

with $b_j=0$ for $j>n_b$ and $a_{j-l}=0$ for $j-l>n_a$.

Define the parameter vector of OE model in Eq. 18 as

$$\boldsymbol{\theta}=[a_1,a_2,\cdots,a_{n_a},b_0,b_1,\cdots,b_{n_b}]^{\mathrm{T}}\in\mathbb{R}^{(n_a+n_b+1)\times 1}$$

Equation 20 can be expanded into the following vector form

$$\boldsymbol{\vartheta}=[-\boldsymbol{S}_a,\boldsymbol{S}_b]\ \boldsymbol{\theta} \quad (21)$$

where $\boldsymbol{S}_a\in\mathbb{R}^{(n_f+1)\times n_a}$ and $\boldsymbol{S}_b\in\mathbb{R}^{(n_f+1)\times(n_b+1)}$ are matrices formed by

$$\boldsymbol{S}_a=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ f_0 & 0 & \cdots & 0 \\ f_1 & f_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ f_{n_a-1} & f_{n_a-2} & \cdots & f_0 \\ \vdots & \vdots & & \vdots \\ f_{n_f-1} & f_{n_f-2} & \cdots & f_{n_f-n_a} \end{bmatrix}, \quad \boldsymbol{S}_b=\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \vdots & & \ddots & 1 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

Applying least-squares (LS) algorithm to Eq. 21, the parameter estimate $\hat{\boldsymbol{\theta}}$ of the OE model can be directly extracted from the parameter estimate $\hat{\boldsymbol{\vartheta}}$ of the FIR model, given by

$$\hat{\boldsymbol{\theta}}=\{\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\}^{-1}\boldsymbol{S}^{\mathrm{T}}\hat{\boldsymbol{\vartheta}} \quad (22)$$

where $\boldsymbol{S}=[-\hat{\boldsymbol{S}}_a,\boldsymbol{S}_b]$ and $\hat{\boldsymbol{S}}_a$ is formed by replacing f_j in $\boldsymbol{S}_a$ with its estimate $\hat{f}_j$ given in $\hat{\boldsymbol{\vartheta}}$.

Although this direct method is convenient for implementation, it is sensitive to the measurement noise. One possible reason is that the uncertainty of the FIR model parameters increases as the noise level increases. Using the estimated FIR model to recover the OE model based on Eq. 22 may amplify the errors. Therefore, we will provide an alternative method that is still under the framework of EM algorithm for recovery of the OE model based on the identified FIR model. Using the auxiliary model identification principle, the obtained FIR model is adopted to generate the estimates of the noise-free outputs, and then EM algorithm is applied again to estimate the OE model parameters along with the random delays based on the MR input–output data and the noise-free output estimates. Since the available MR data is repeatedly used in recovery of the OE model, the proposed method can effectively improve the estimation precision.

The OE model in Eq. 18 can be rewritten as

$$x_k = \boldsymbol{\varphi}_k \boldsymbol{\theta} \tag{23}$$

where $\boldsymbol{\varphi}_k$ is a new information vector defined as

$$\boldsymbol{\varphi}_k = [-x_{k-1}, -x_{k-2}, \cdots, -x_{k-n_a}, u_k, u_{k-1}, \cdots, u_{k-n_b}] \tag{24}$$

Substituting Eq. 23 into Eq. 19 yields

$$y_{T_i} = \boldsymbol{\varphi}_{T_i-\lambda_i} \boldsymbol{\theta} + v_{T_i} \tag{25}$$

Replacing $\boldsymbol{\vartheta}$ in Eq. 4 with its estimate $\hat{\boldsymbol{\vartheta}}$, the noise-free output can be estimated by

$$\hat{x}_k = \boldsymbol{\psi}_k \hat{\boldsymbol{\vartheta}} \tag{26}$$

Based on $\hat{x}_k$, the estimate of $\boldsymbol{\varphi}_k$ in Eq. 24 is formed by

$$\hat{\boldsymbol{\varphi}}_k = [-\hat{x}_{k-1}, -\hat{x}_{k-2}, \cdots, -\hat{x}_{k-n_a}, u_k, u_{k-1}, \cdots, u_{k-n_b}] \tag{27}$$

Using $\hat{\boldsymbol{\varphi}}_{T_i-\lambda_i}$ to replace $\boldsymbol{\varphi}_{T_i-\lambda_i}$ in Eq. 25, we have

$$y_{T_i} = \hat{\boldsymbol{\varphi}}_{T_i-\lambda_i} \boldsymbol{\theta} + v_{T_i} \tag{28}$$

Similar to Eq. 15, the Q function of the EM algorithm for identification of $\boldsymbol{\theta}$ is given by

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^h) = \log C + \sum_{i=1}^{N}\sum_{j=0}^{q} w_{ij'}\left[-\log\sqrt{2\pi}\hat{\sigma}_v - \frac{1}{2\hat{\sigma}_v^2}\left[y_{T_i} - \hat{\boldsymbol{\varphi}}_{T_i-\lambda_i}\boldsymbol{\theta}\right]^2\right] + \sum_{i=1}^{N}\sum_{j=0}^{q} w_{ij}{}'\log\left[p(\lambda_i=j)\right] \tag{29}$$

where $\hat{\sigma}_v^2$ is the estimate of the noise variance given in the previous section; the conditional probability $w_{ij}{}'$ is derived as

$$w_{ij'} = \frac{p\left(y_{T_i}|\hat{\boldsymbol{\varphi}}_{T_i-j}, \lambda_i=j, \boldsymbol{\theta}^h, \hat{\sigma}_v^2\right)p(\lambda_i=j)}{\sum_{j=0}^{q} p\left(y_{T_i}|\hat{\boldsymbol{\varphi}}_{T_i-j}, \lambda_i=j, \boldsymbol{\theta}^h, \hat{\sigma}_v^2\right)p(\lambda_i=j)} \tag{30}$$

Then the delays are re-estimated by

$$\hat{\lambda}_i{}' = \arg\max_j w'_{ij}, j=0,1,\cdots,q, i=1,2,\cdots,N$$

The new parameter estimate $\boldsymbol{\theta}^{h+1}$ is obtained through maximization of the Q function in Eq. 29, which is

$$\boldsymbol{\theta}^{h+1} = \left\{\sum_{i=1}^{N}\sum_{j=0}^{q} w'_{ij} * \hat{\boldsymbol{\varphi}}^{\mathrm{T}}_{T_i-j}\hat{\boldsymbol{\varphi}}_{T_i-j}\right\}^{-1}\left\{\sum_{i=1}^{N}\sum_{j=0}^{q} w'_{ij} * \hat{\boldsymbol{\varphi}}^{\mathrm{T}}_{T_i-j} y_{T_i}\right\} \tag{31}$$

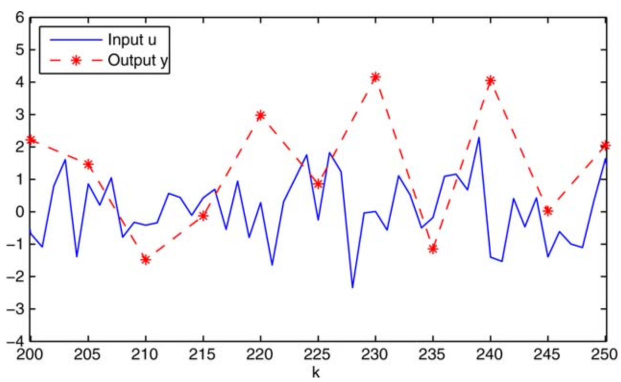

Figure 1. The MR sampled inputs and outputs (numerical example).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

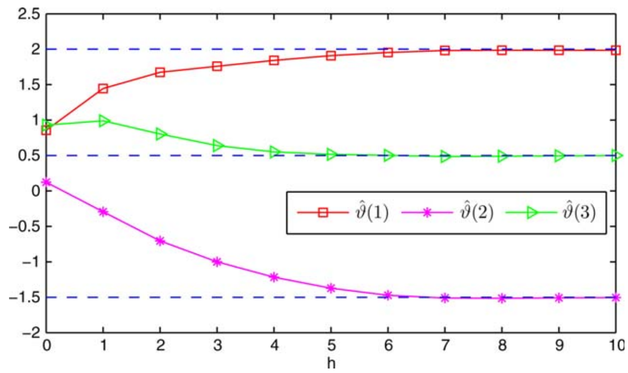

Figure 2. The EM estimations of FIR model parameters vs. iteration *h*.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

In this way, the parameter vector $\boldsymbol{\theta}$ of the OE model along with the delays $\{\lambda_i\}$ are estimated based on the fast-rate input $\{u_k\}$, noise-free output estimate $\{\hat{x}_k\}$, and slow-rate output $\{y_{T_i}\}$.

Simulations

A numerical simulation example

Consider the following MR FIR system

$$x_k = 2u_k - 1.5u_{k-1} + 0.5u_{k-2} \tag{32}$$
$$y_{T_i} = x_{T_i-\lambda_i} + v_{T_i}$$

where the fast-rate input sequence $\{u_k\}$ is generated from Gaussian distribution $N(0, \sigma_u^2)$ with $\sigma_u^2=1$; the slow-rate output $\{y_{T_i}\}$ is available at time instant $T_i \cdot \Delta t (T_i = 5i)$ with random time delay $\lambda_i \cdot \Delta t (\lambda_i \in [0,4])$; the variance of the measurement noise $\{v_{T_i}\}$ is $\sigma_v^2=0.01$; thus the noise to signal ratio is $\delta_{\mathrm{ns}} = \sqrt{\sigma_v^2/\sigma_u^2} = 10\%$. In simulation, $L = 500$ fast-rate inputs and $N = 100$ slow-rate outputs are collected for system identification, part of which is shown in Figure 1. Base on the available MR data, the parameter vector of the FIR model to be identified is $\boldsymbol{\vartheta} = [2, -1.5, 0.5]^{\mathrm{T}}$, and the noise variance to be estimated is $\sigma_v^2 = 0.01$.

Applying the proposed EM algorithm with a randomly generated initial guess to identify the unknown parameters, the estimated FIR model parameters and the estimated noise variance vs. iteration h are shown in Figures 2 and 3, respectively. From these two figures, it can be observed that the proposed EM algorithm has good identification performance since the estimated parameters approach the real ones after a few iterations.

The LS algorithm is also applied for parameter estimation, the comparison results are listed in Table 1. From this table, it can be observed that the LS estimation is inaccurate if the random delays are simply neglected, while the EM algorithm considering random delays can provide much better estimation, and its performance is almost comparable with the LS algorithm when the delays are completely known.

Based on the EM parameter estimation $\hat{\boldsymbol{\vartheta}} = [1.9861, -1.5043, 0.4998]^{\mathrm{T}}$, the delay estimation is shown in Figure 4, where the circle and the cross indicate the correct and the incorrect estimations, respectively. The accuracy of delay estimation vs. noise to signal ratio δ_{ns} is shown in Figure 5, which demonstrates that the uncertainty of delay estimation is mainly dependent on the measurement noise. Not

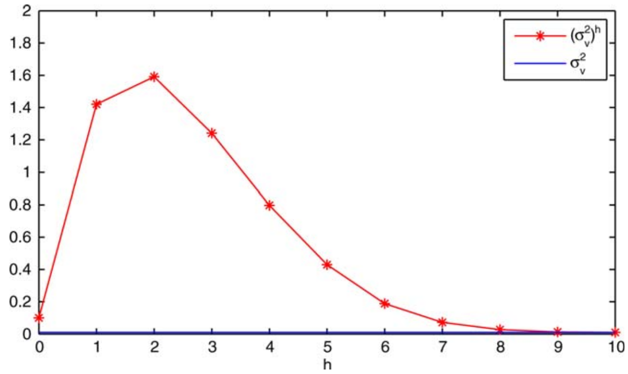

Figure 3. The EM estimations of noise variance vs. iteration *h*.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table 1. Parameter Estimates and Relative Errors of Different Methods

True parameter	EM (10 iterations)	LS (neglected delays)	LS (known delays)
$\boldsymbol{\theta}(1)=2$	1.9861	0.3431	1.9924
$\boldsymbol{\theta}(2)=-1.5$	−1.5043	0.0280	−1.5001
$\boldsymbol{\theta}(3)=0.5$	0.4998	0.0173	0.4997
Relative error	0.0031%	81.7401%	0.0009%

surprisingly, the higher the noise level, the lower the estimation precision.

A continuous stirred tank heater (CSTH)

A simplified schematic diagram of a continuous stirred tank heater (CSTH) is shown in Figure 6, where the cold water flowing into the tank is heated by passing the steam through the coil. Four metal sheathed thermocouples are equipped for measuring the temperature of the outlet hot water, which are evenly spaced along the outflow pipe (i.e., A, B, C, and D four points). It will take 4 s for the water to flow from one point to the next (e.g., A to B), leading to the transmission time delay. The relationship between the steam valve position (i.e., input u) and the temperature measured at A (i.e., noise-free output x) is of interest. Thornhill et al. (2008)[23] derived the following continuous transfer function model under the operating conditions shown in Table 2.

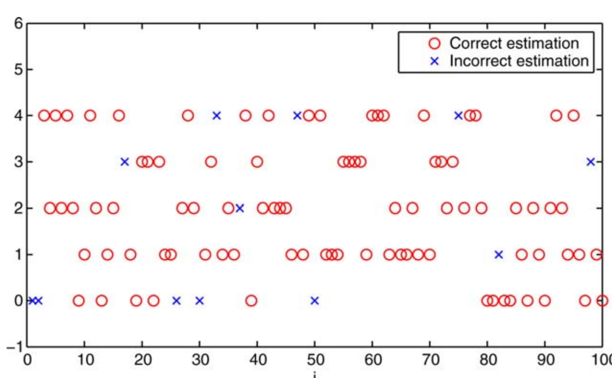

Figure 4. The EM estimations of delays.

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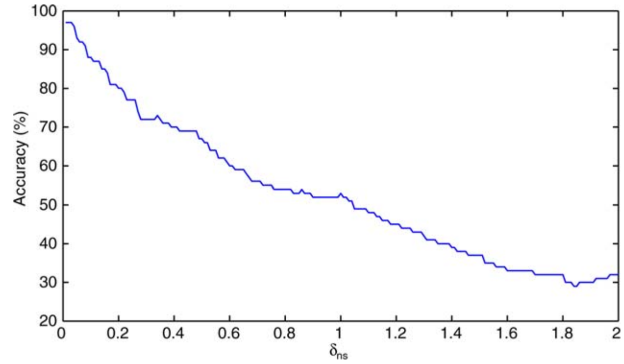

Figure 5. The accuracy of delay estimation vs. δ_{ns}.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

$$x(s)=\frac{1.5867\times10^{-2}e^{-8s}}{s+2.732\times10^{-2}}u(s) \tag{33}$$

Taking the sampling period Δt as 4 s, the discrete transfer function model corresponding to Eq. 33 is derived as

$$x_k=\frac{0.06012z^{-1}}{1-0.8965z^{-1}}z^{-2}u_k \tag{34}$$

Because of the transmission delay, the observation y_k at time $k\cdot\Delta t$ measured at A, B, C, and D should correspond to x_k, x_{k-1}, x_{k-2}, and x_{k-3}, respectively.

To simulate the MR system with random sampling delays, we assume that the fast-rate input u_k is sampled with interval Δt; the slow-rate output y_{T_i} is randomly provided by one of the four thermocouples with time interval $4\cdot\Delta t$. Thus, the measurement equation can be expressed as

$$y_{T_i}=x_{T_i-\lambda_i}+v_{T_i}$$

where λ_i is the random delay ranging from 0 to 3, and v_{T_i} is the measurement noise.

Four thousand input and 1000 output data are collected for identification, part of which is shown in Figure 7, where the input sequence u_k is generated from Gaussian distribution $N(0,1)$, and the output measurement noise v_{T_i} follows Gaussian distribution $N(0,0.01)$.

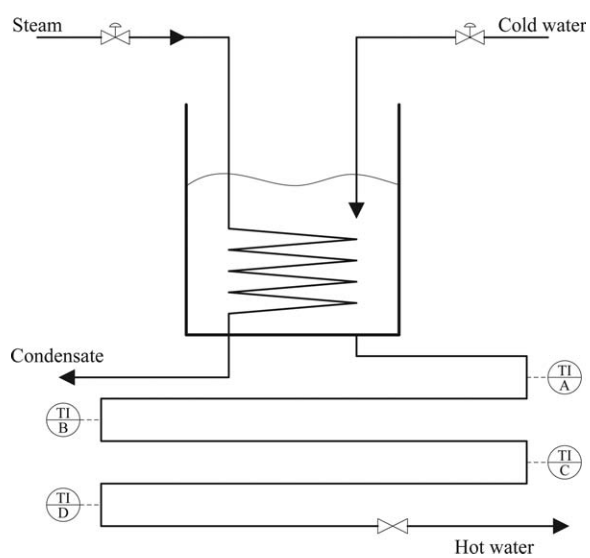

Figure 6. Schematic diagram of the CSTH system.

Table 2. Nominal Operating Conditions of the CSTH System

Variable	Value
Level	12 mA (20.48 cm)
Cold valve	12.96 mA (9.038×10^{-5} m^3/s)
Steam valve	12.57 mA
Temperature	10.5 mA (42.52°C)

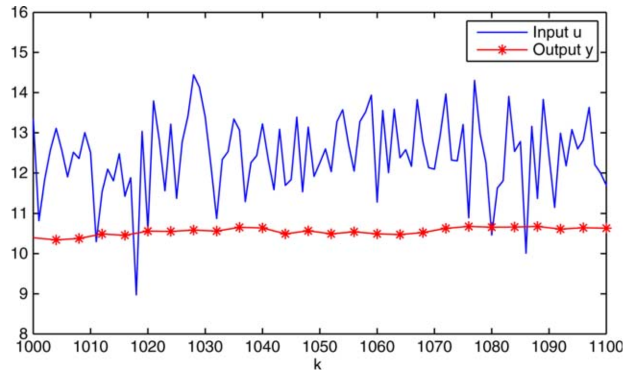

Figure 7. The MR sampled inputs and outputs (CSTH example).

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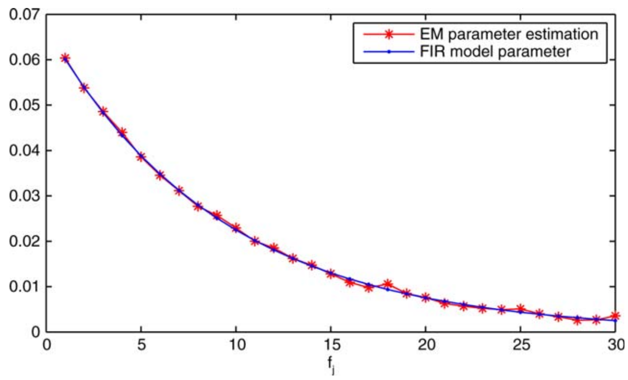

Figure 8. The EM estimations of FIR model parameters after convergence (CSTH example).

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An FIR model of relatively higher order should be used to approximate the OE model in Eq. 34 because the pole is close to the unit circle in this example. Letting $n_f=30$, the proposed EM algorithm with a randomly generated initial guess is applied to estimate the parameter $\boldsymbol{\vartheta}$ of the approximated FIR model. The parameter estimation after convergence $\hat{\boldsymbol{\vartheta}}$ is shown in Figure 8.

Based on the identified FIR model parameters $\hat{\boldsymbol{\vartheta}}$, the OE model parameters $\boldsymbol{\theta}$ can be directly recovered using the LS algorithm. The estimation results referred to as FIR–LS under different noise variance σ_v^2 are shown in Table 3, in comparison with the FIR–EM estimations which are given by the EM algorithm using the FIR model predictions. From this table, it can be observed that these two methods both have excellent estimation performance when the noise level is low. However, the performance of FIR–LS dramatically deteriorates as the noise variance increases, while FIR–EM is better than FIR–LS even when the noise level is high. For $\sigma_v^2=0.01$, FIR–EM estimations of the OE model parameters vs. iteration h are shown in Figure 9. As it is expected, the parameter estimation approaches the real value after several iterations.

Figure 10 compares the predictions of the estimated OE model with the noise-free outputs and the sampled noisy outputs, which demonstrates the effectiveness of the proposed EM algorithm. The model prediction errors under different noise level are compared in Table 4. It shows that the prediction performance of the estimated OE model improves as the noise level decreases. Since the slowly sampled data are often measured through laboratory analysis, the associated noise under laboratory environment is typically small. Thus, the proposed EM algorithm for identification of the MR OE model will be promising in practice.

Experimental evaluation: a three-tank system

An experiment is conducted on a three-tank system[19,24] to further verify the effectiveness of the proposed identification algorithm. Figure 11 shows the configuration of the three-tank system, where the water is pumped into the top tank by a DC pump, and then flows through each tank under the influence of gravity. The valves located at the bottom of each tank are used to control the outflow rate. On the assumption that the laminar outflow is an ideal fluid, the dynamics of the three-tank system can be described as follows[19]

$$\frac{dH_1}{dt}=\frac{1}{\beta_1(H_1)}q-\frac{1}{\beta_1(H_1)}C_1\sqrt{H_1} \tag{35}$$

$$\frac{dH_2}{dt}=\frac{1}{\beta_2(H_2)}C_1\sqrt{H_1}-\frac{1}{\beta_2(H_2)}C_2\sqrt{H_2} \tag{36}$$

$$\frac{dH_3}{dt}=\frac{1}{\beta_3(H_3)}C_2\sqrt{H_2}-\frac{1}{\beta_3(H_3)}C_3\sqrt{H_3} \tag{37}$$

where q is the inflow to the upper tank; H_i, β_i, and C_i are the water level, cross section area, and resistance of the output orifice of the ith tank $(i=1,2,3)$, respectively.

The inflow q and the second tank water level H_2 are considered as the system input u and output y, respectively. Let the system operate at a steady-state condition, where $u=0.34$, $y=8.97$cm. Then a random binary signal with level $[-0.03\quad 0.03]$ is added to the steady-state input to stimulate the system. The experimental input and output data are shown in Figure 12, where the sampling interval is 15 s.

Assume that the system output is available every minute with a random time delay ranging from 0 s, 15 s, 30 s, and

Table 3. Parameter Estimates and Relative Errors Under Different Noise Levels

	$\sigma_v^2=0.01$		$\sigma_v^2=0.04$		$\sigma_v^2=0.25$	
True parameter	FIR–EM	FIR–LS	FIR–EM	FIR–LS	FIR–EM	FIR–LS
$\boldsymbol{\theta}(1)=-0.8965$	−0.8972	−0.8960	−0.8904	−0.8824	−0.9365	−0.5738
$\boldsymbol{\theta}(2)=-0.06012$	0.06188	0.06066	0.06171	0.06066	0.06188	0.05684
Relative error	4.4953×10^{-6}	3.8711×10^{-7}	4.8949×10^{-5}	2.4668×10^{-4}	0.0024	0.1290

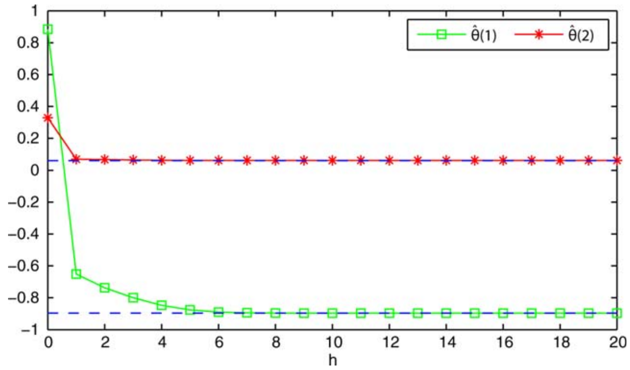

Figure 9. The FIR–EM estimations of OE model parameters vs. iteration *h* (CSTH example).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

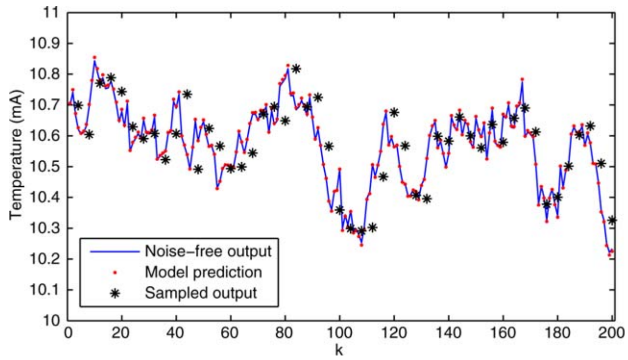

Figure 10. Comparison of model predictions with noise-free outputs $(\sigma_v^2=0.01)$.

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45 s. The resulted MR data are divided into two sets, where the first 400 fast-rate input and 100 slow-rate output data are used for self-validation, and the rest are used for cross-validation, respectively. Applying the proposed EM algorithm to the self-validation dataset, the estimated model is given by

$$y_k=\frac{0.1088z^{-2}+0.0476z^{-3}}{1-0.4872z^{-1}-0.3409z^{-2}}u_k \tag{38}$$

The self-validation results and the cross-validation results of the estimated model are shown in Figures 13 and 14, respectively. The model prediction shows a good fitness with the measured output, which demonstrate that the proposed EM algorithm is effective in the identification of MR system with random sampling delays.

Conclusions

This article considers identification of MR systems with unknown random delays, which includes conventional non-uniformly sampled-data systems and dual-rate systems as special cases. Due to the impact of random delay, the irregularly sampled output cannot appropriately indicate current noise-free output even with no measurement noise. Thus traditional identification methods like LS algorithm fail to identify such MR systems if the uncertain delay problem is overlooked. To address this challenge, an EM algorithm is applied in this article to identify the MR FIR model, in which unknown delays are treated as hidden states and are estimated along with the model parameters. The proposed EM algorithm has been evaluated in a simulation example and the obtained results confirm that the algorithm is effective with high estimation accuracy and fast convergence rate.

OE or transfer function model has extensive applications in development of advanced control. However, due to

Table 4. Model Prediction Errors Under Different Noise Levels

	$\sigma_v^2=0.01$	$\sigma_v^2=0.04$	$\sigma_v^2=0.25$
Prediction error	1.9211×10^{-5}	2.3286×10^{-5}	0.0013

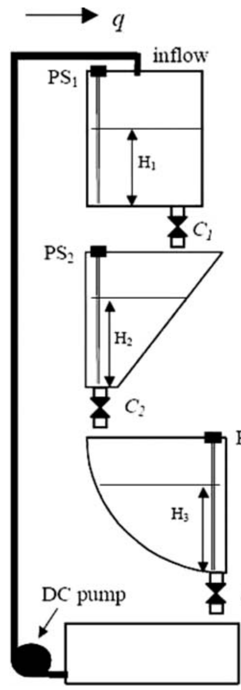
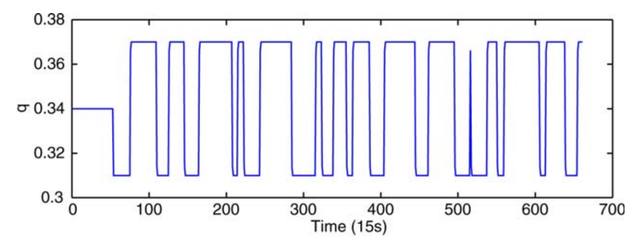
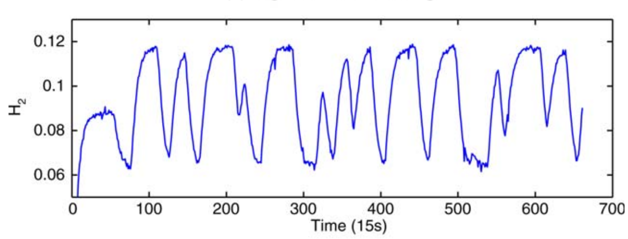


Figure 11. The configuration of the three-tank system.



(a) Input u: the inflow q



(b) Output y: the second tank water level H_2

Figure 12. The experimental input and output data of the three-tank system.

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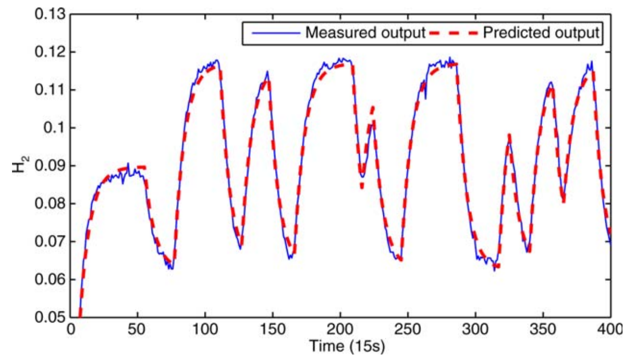

Figure 13. Self-validation results of the estimated model.

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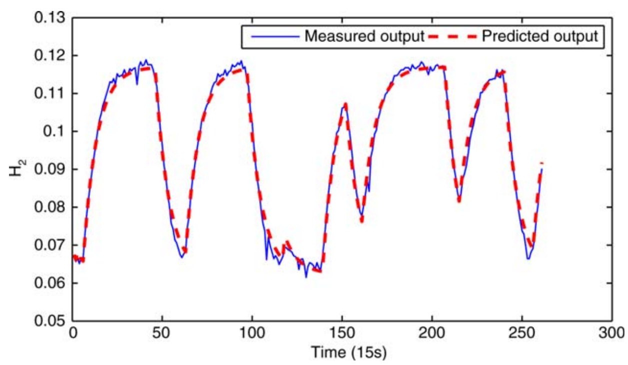

Figure 14. Cross-validation results of the estimated model.

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coexistence of unknown noise-free outputs and random delays, it is difficult to directly identify an OE model based on the available MR data. Therefore, recovery of the OE model based on the estimated FIR model is further investigated in this article. The effectiveness of the proposed methods is demonstrated through a simulated CSTH example and an experimental three-tank system.

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